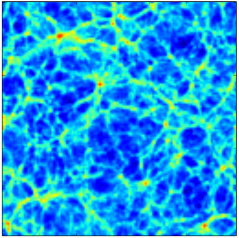

Bayesian inference of cosmic density fields from biased tracers

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September 2, 2014

Overview

1. Motivation
2. Method
3. Numerical Tests
4. Conclusions

Overview

Motivation

The dark matter density field contains all cosmological information of interest:

- $\delta_{\mathrm{M}}(\boldsymbol{r}) \rightarrow P(\boldsymbol{k})$(e.g. reconstruction of BAO), cosmological parameters, $B(\boldsymbol{k}_1, \boldsymbol{k}_2)$..., $\Phi(\boldsymbol{r})$, ISW
- $\boldsymbol{v}(\boldsymbol{r}) \rightarrow$ growth rate $\sigma_8 f(z)$, RSD, kinetic SZ-effect...
- $\delta_{\mathrm{M}}(\boldsymbol{q}) \rightarrow$ Initial Conditions, e.g. Constraint Simulations

So we need unbiased reconstructions of the dark matter density field

Motivation

What is our input?

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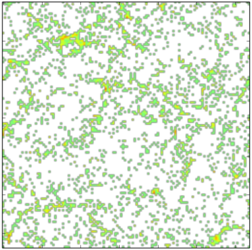
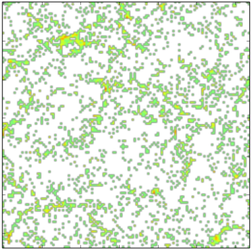


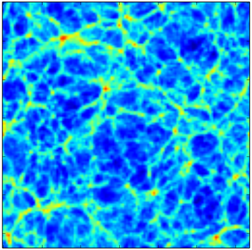
BDM Halos of the Bolshoi catalogue
on 128^3 grid

Motivation

What is our input?



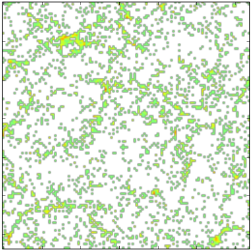
BDM Halos of the Bolshoi catalogue on 128^3 grid



Dark matter distribution of Bolshoi catalogue on 128^3 grid

Challenges

We have to deal with several challenging tasks



I Number density (Shot noise), completeness, mask, selection function

II Bias (stochastic, non-linear, non-local)

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Previous Works

Methods used in Cosmology in the past to infer dark matter density field:

1. Poisson model with linear bias ⟶ Over-simplified models
   Zaroubi+95, Erdogdu+96, Kitaura+09, Jasche+10 etc.
2. Halo model reconstruction ⟶ requires very complete data and neglets stochastic bias
   Wang+09,11; Muñoz-Cuartas+11

A fundamental description of the stochastic nature of the bias is missing in inference algorithms!

Method

Our method relies on Bayesian statistics:
Posterior $\propto$ **Prior** $\times$ **Likelihood**

1. Posterior $\mathcal{P}(\boldsymbol{\delta}_{\rm M}|\boldsymbol{N}_{\rm G}, I)$,
2. Prior $\pi(\boldsymbol{\delta}_{\rm M}|\{p_{\rm c}\}, I)$,
3. Likelihood $\mathcal{L}(\boldsymbol{N}_{\rm G}|\mathcal{B}(\boldsymbol{N}_{\rm G}|\boldsymbol{\delta}_{\rm M}), I)$,
4. $\mathcal{B}(\boldsymbol{N}_{\rm G}|\boldsymbol{\delta}_{\rm M})$ is our bias model that connects the prior to the likelihood, including stochastic and non-linear components

Prior

- Prior $\pi(\boldsymbol{\delta}_{\mathrm{M}}|\{p_{\mathrm{c}}\})$ represents knowledge of dark matter distribution in this framework
- Any structure formation model (LPT...) can be considered
- We use the lognormal assumption:
  Define s as logarithmic transform of density field:

$$\boldsymbol{s} = \log(1 + \boldsymbol{\delta}_{\mathrm{M}}) - \boldsymbol{\mu}, \quad \boldsymbol{\mu} = \langle \log(1 + \boldsymbol{\delta}_{\mathrm{M}}) \rangle$$

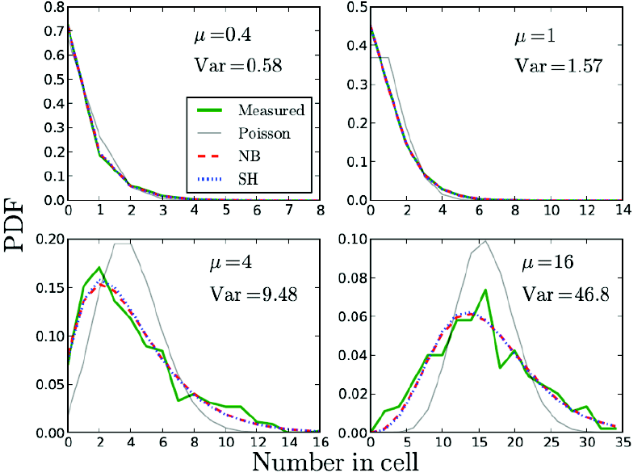
- s is Gaussian (solution of continuity equation in comoving frame Coles&Jones91)
- Linearization/Gaussianization optimal for cosmological parameter estimation (Neyrinck+11)
- Good approximation of sufficient statistics (Carron&Szapudi13)

Likelihood

- Reminder: Likelihood models statistical nature of data $\mathcal{L} = \mathcal{L}(\boldsymbol{N}_{\rm G}|\boldsymbol{\lambda}, I)$
- We focus on over-dispersion of galaxy counts in cells, studied by Peebles80, Somerville+01...Baldauf+14
- The NB $f_{\rm NB}$ and the SH $f_{\rm SH}$ (Saslaw&Hamilton84) distributions do model overdispersion with additional parameter β, b, Poisson does not:

$$
\begin{aligned}
f_{\rm P}(\lambda, N) &= \frac{{\rm e}^{-\lambda}\lambda^N}{N!}\,, \\
f_{\rm NB}(\lambda, N, \beta) &= \frac{\lambda^N}{N!}\frac{\Gamma(\beta+N)}{\Gamma(\beta)(\beta+\lambda)^N}\frac{1}{\left(1+\frac{\lambda}{\beta}\right)^{\beta}}\,, \\
f_{\rm SH}(\lambda, N, b) &= \frac{\lambda}{N!}{\rm e}^{-\lambda(1-b)-bN}(1-b)\left[\lambda(1-b)+bN\right]^{N-1}
\end{aligned}
$$

Likelihood



Neyrinck+13, based on MIP simulations Aragon-Calvo13

Bias: Link between Prior and Likelihood

- Link prior (dark matter field) and likelihood (galaxy counts) with bias function $\mathcal{B}(\boldsymbol{N}_{\mathrm{G}}|\boldsymbol{\delta}_{\mathrm{M}})$
- $\mathcal{B}(\boldsymbol{N}_{\mathrm{G}}|\boldsymbol{\delta}_{\mathrm{M}})$ relates to the expected number counts $\boldsymbol{\lambda}$ according to biasing parameters $\{p_{\mathrm{SB}}\}$:
  $\mathcal{L}(\boldsymbol{N}_{\mathrm{G}}|\mathcal{B}(\boldsymbol{N}_{\mathrm{G}}|\boldsymbol{\delta}_{\mathrm{M}})) = \mathcal{L}(\boldsymbol{N}_{\mathrm{G}}|\boldsymbol{\lambda}, \{p_{\mathrm{SB}}\})$
- General form is non-linear, scale-dependent and non-local
- Drawing discrete numbers $\boldsymbol{N}_{\mathrm{G}}$ from function $\mathcal{B}(\boldsymbol{N}_{\mathrm{G}}|\boldsymbol{\delta}_{\mathrm{M}})$ implies stochasticity

Bias

Attempts to express bias relation in a pertubation series:

- Fry&Gaztañaga93:

$$\boldsymbol{\rho}_{\rm h} \propto \sum_i a_i \boldsymbol{\delta}_{{\rm M}i}$$

- Cen&Ostriker93:

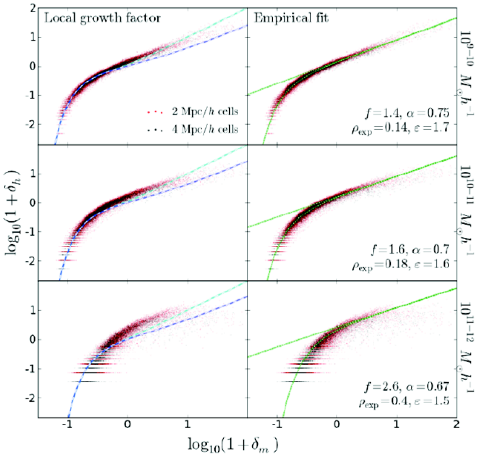
$$\boldsymbol{\rho}_{\rm h} \propto \exp\left[\sum_i b_i \log(1+\boldsymbol{\delta}_{{\rm M}i})\right]$$

- Kitaura,Yepes&Prada14; Neyrinck+14:

$$\boldsymbol{\rho}_{\rm h} \propto \boldsymbol{\rho}_{\rm M}^{\alpha} \exp\left[-\left(\frac{\rho_{\rm M}}{\rho_\epsilon}\right)^{\epsilon}\right]$$

- We use : $\lambda_i \equiv \langle \rho_{{\rm G}i} \rangle = f_{\bar{N}} w_i (\rho_{{\rm M}i})^{\alpha} \exp\left[-\left(\frac{\rho_{{\rm M}i}}{\rho_\epsilon}\right)^{\epsilon}\right]$

Bias relation



Neyrinck+13, Aragon-Calvo13

Sampling from the Posterior

- We want to sample from the full posterior
- We use Hamiltonian sampling Duane+87, Neal93 to sample from the posterior $\mathcal{P}(\boldsymbol{s})$

$$\begin{aligned} \mathcal{H}(\boldsymbol{s},\boldsymbol{p}) &= U(\boldsymbol{s}) + K(\boldsymbol{p})\,, \\ U(\boldsymbol{s}) &= -\ln \mathcal{P}(\boldsymbol{s}) \end{aligned}$$

- The kinetic term is given $K(\boldsymbol{p}) = \sum_{i,j} \frac{1}{2} p_i M_{ij}^{-1} p_j$.
- So $\mathcal{P}(\boldsymbol{s})$ can be infered from

$$\exp(-\mathcal{H}) = \mathcal{P}(\boldsymbol{s}) \cdot \exp\left(-\sum_{i,j} \frac{1}{2} p_i M_{ij}^{-1} p_j\right).$$

Sampling

- Evolution of system with Hamiltonian equations of motion

$$\frac{dx_i}{dt} = \frac{\partial \mathcal{H}}{\partial \boldsymbol{p}}$$
$$\frac{dp_i}{dt} = -\frac{\partial \mathcal{H}}{\partial \boldsymbol{s}}$$

- Accept iteration if:

$$P_{\mathrm{Accept}} = \min\left[1, \exp\left(-\mathcal{H}(\boldsymbol{s_1}, \boldsymbol{p_1}) + \mathcal{H}(\boldsymbol{s_0}, \boldsymbol{p_0})\right)\right]$$

- We need therefore $-\ln \mathcal{P}(\boldsymbol{s})$ and its gradient w.r.t. signal $\boldsymbol{s}$

AIP

Sampling

$$P(\delta_M|\mathbf{N}, S(\{p_C\})) = \frac{1}{\sqrt{(2\pi)^{N_C}\det(\mathbf{S})}} \prod_{l=1}^{N_C} \frac{1}{1+\delta_M^l} \tag{1}$$

$$\times \exp\left(-\frac{1}{2}\sum_{ij}\left[(\ln(1+\delta_M^i)-\mu^i)S_{ij}^{-1}(\ln(1+\delta_M^j)-\mu^j)\right]\right)$$

$$\times \prod_{l=1}^{N_C}\left(\frac{f_{\bar{N}}w^l(1+\delta_{\rm M}^l)^{\alpha}\mathrm{e}^{\left[-\left(\frac{1+\delta_{\rm M}^l}{\rho_\epsilon}\right)^{\epsilon}\right]}\Gamma(\beta+N^l)}{N^l!\,\Gamma(\beta)\left(\beta+f_{\bar{N}}w^l(1+\delta_{\rm M}^l)^{\alpha}\mathrm{e}^{\left[-\left(\frac{1+\delta_{\rm M}^l}{\rho_\epsilon}\right)^{\epsilon}\right]}\right)^{N^l}\left(1+\frac{f_{\bar{N}}w^l(1+\delta_M^l)^{\alpha}\mathrm{e}^{\left[-\left(\frac{1+\delta_{\rm M}^l}{\rho_\epsilon}\right)^{\epsilon}\right]}}{\beta}\right)^{\beta}}\right).$$

Sampling

$$-\frac{\partial}{\partial s_i}\ln\mathcal{P}(\delta_\mathrm{M}|N_G) = -\frac{\partial}{\partial s_i}\ln\pi(\delta_\mathrm{M}|\{p_C\}) - \frac{\partial}{\partial s_i}\ln\mathcal{L}(N_G|\lambda)$$

$$\begin{aligned}
-\frac{\partial}{\partial s_i}\ln\pi &= \frac{1}{2}\sum_{ij}(\delta_{ik}(S^{-1}_{ij}s_j) + \delta_{jk}(s_i S^{-1}_{ij})) \\
&= \frac{1}{2}\left[\sum_j (S^{-1}_{kj}s_j) + \sum_i (s_i S^{-1}_{ik})\right] \\
-\frac{\partial}{\partial s_i}\ln\pi &= \mathbf{S}^{-1}\mathbf{s}
\end{aligned}$$

$$\begin{aligned}
-\ln\mathcal{L}_\mathrm{NB} &= \sum_i(-N_i\ln\lambda_i + N_i\ln(\beta+\lambda_i) + \beta\ln(1+\lambda_i/\beta) - c) \\
-\ln\mathcal{L}_\mathrm{SH} &= \sum_i(-\ln\lambda_i + \lambda_i(1-b) - (N_i-1)\times\ln(\lambda_i(1-b)+N_i b) - c)
\end{aligned}$$

$$\begin{aligned}
\left(\frac{\partial s_i}{\partial\delta_j}\right)^{-1} &= 1+\delta_j \\
\frac{\partial\lambda_k}{\partial\delta_j} &= \frac{\alpha\lambda_j}{1+\delta_j} - \frac{\epsilon\lambda_j}{1+\delta_j}\left(\frac{1+\delta_j}{\rho_\epsilon}\right)^\epsilon \\
-\frac{\partial\ln\mathcal{L}_\mathrm{NB}}{\partial\lambda_k} &= -\frac{N_k}{\lambda_k} - \frac{N_k}{\beta+\lambda_k} + \frac{1}{1+\frac{\lambda_k}{\beta}}
\end{aligned}$$

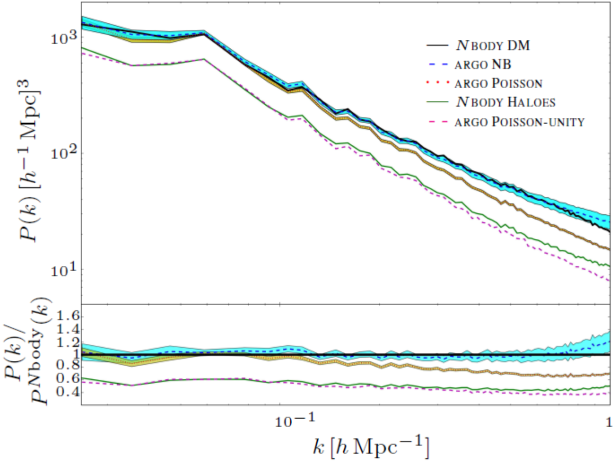
Overview

AIP

- We use the Bolshoi (Klypin+11) 250 h^{-1} Mpc dark Matter Simulation containing 2048^3 particles with WMAP5/7 compatible Cosmology.
- We also use a Halo catalogue based on Bolshoi calculated with BDM
- Subset taken selecting $2 \cdot 10^5$ halos to demonstrate ARGO'S performance
- http://www.multidark.org/MultiDark/
- http://www.cosmosim.org/

Testing ARGO

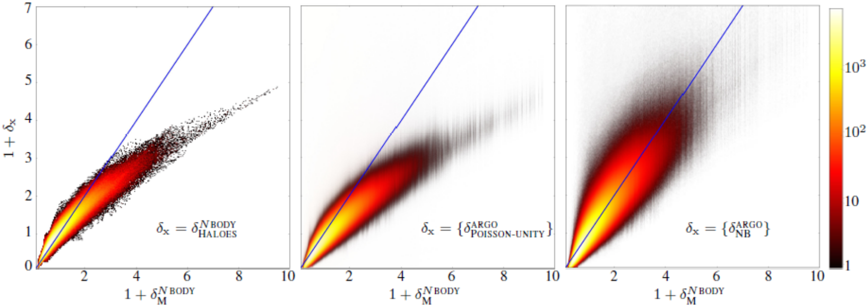
Metin Ata+14 arXiv:1408.2566



- We run ARGO (Kitaura+12) with different configurations
- Each output of ARGO is a full catalogue
- We compare outputs to *true* dark matter distribution

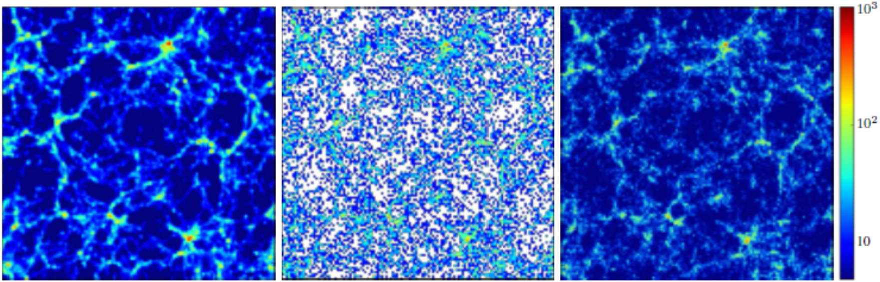
Testing ARGO

Metin Ata+14 arXiv:1408.2566



testing ARGO

Metin Ata+14 arXiv:1408.2566



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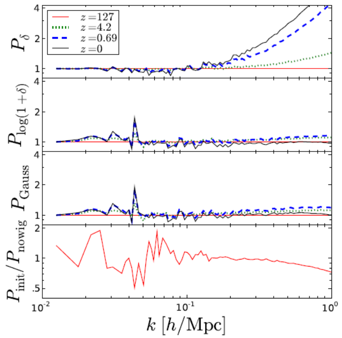
Conclusions and future work

- Developed bayesian reconstruction framework to deal with over-dispersed distributed data and non-linear, stochastic bias
- Implemented this into the ARGO-code
- Tested with Bolshoi Dark Matter simulation
- Created unbiased dark matter distributions in 2point-statistics

TO DO:

- Implementing RSD
- Full framework to be applied to BOSS survey

Appendix A



Neyrinck+09

- $\pi(\boldsymbol{\delta}_{\mathrm{M}}|\mathbf{S}) = \frac{1}{\sqrt{(2\pi)^{N_{\mathrm{C}}}\det(\mathbf{S})}} \exp\left(-\frac{1}{2}\boldsymbol{s}^{\dagger}\mathbf{S}^{-1}\boldsymbol{s}\right)$
- The covariance matrix $\mathbf{S} \equiv \langle \boldsymbol{s}^{\dagger}\boldsymbol{s}\rangle$ (or power spectrum in Fourier-space)
- If $\boldsymbol{\delta}_{\mathrm{M}} \ll 1 \rightarrow \pi$ is Gaussian